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Monthly Progress Report P-B1980-5

HERO SUPPORTING STUDIES

by

Norman P. Faunce
Paul F. Mohrbach

November 1 to November 30, 1962

Prepared for

U. S. NAVAL WEAPONS LABORATORY
Dahlgren, Virginia

Contract No. N178-8102

THE FRANKLIN INSTITUTE
LABORATORIES FOR RESEARCH AND DEVELOPMENT
PHILADELPHIA PENNSYLVANIA

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ABSTRACT

The reduction of voltage maximum and voltage minimum readings to indicate power delivered to an arbitrary termination is the subject of recent study. In one experiment, power delivered to a MARK 1 MOD 0 squib determined in this manner is either too high, or the squib plug is more lossy at 1 Gc than was supposed. Another test using a uniform system indicates that successive power readings made along a length of slotted line serve to indicate the system loss. That this technique leads to reliable estimates of the power reaching the termination is yet uncertain. Except for bad data due to the inclusion of a faulty section of adjustable coaxial line, validation of this conjecture would have occurred this period. More experimental and theoretical work will need be performed.

SUMMARY

Completion of the series of tests for the protected MARK 7 MOD 0 ignition element outlined earlier continues to be the primary objective of this program. However, the greatest deterrent to this effort is the lack of means for reliably measuring RF power at the input of any arbitrary load (any EED) terminating the firing line. Recent experiments have indicated that a technique based upon detecting the voltage maximum and minimum along a section of the firing line may remove this obstacle.

One system incorporating a Hewlett-Packard slotted line was used to determine the power delivered to a MARK 1 MOD 0 plug assembly. Test results for three levels of system input power were examined. The accumulation of data is sketchy and, though the sought-for conclusion may in some sense be extracted from the data, certain discrepancies invalidate it. Results, however, were reasonable enough to suggest a more controlled experiment.

There was some concern felt, because the system was assembled from a mixture of components of varying loss including the termination. This makes the task of theoretical correlation difficult, if not impossible. It was decided therefore to repeat the experiment with a more uniform system terminated in a well-defined load. A system of General Radio hardware throughout was put together, and especially prepared disc resistors were used as loads. Results from these experiments are more definite in demonstrating the technique. However, the inclusion of a faulty section of adjustable line prohibited the system's validation. Nonetheless results are sufficiently encouraging that we shall plan to use this technique for the remaining high frequency tests for the MARK 7 element. Certain pieces of necessary equipment are on order, and the test will commence as soon as we receive them.

Additionally, we have supported this search of power measurement techniques with theoretical investigations. Two analyses dealing with VSWR measurements have been made. One indicates that the point corresponding to the SWR (and the power) computed from adjacent voltage maximum and minimum is located (very nearly) at the voltage minimum point if the VSWR is greater than about 7. A second analysis relates the true VSWR at the termination to the measurement of VSWR on the line. Very small line losses are thus shown to be accountable for sizeable changes in VSWR along the line.

As a by-product of experiments designed to validate this power measuring technique, we have collected evidence indicating that the MARK 1 MOD 0 squib plug may be more lossy than previously supposed. A test to reaffirm an earlier system calibration figure was conducted;

the results were positive. However, recalibrating with an additional piece of coaxial tubing permitted the determination of the loss in the base of the squib termination. To the contrary, other data of equal validity led to the conclusion that there was no loss. Additional work should reveal which conclusion is more nearly correct.

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1. MEASUREMENT OF POWER DELIVERED TO AN ARBITRARY TERMINATION

Tests to determine the sensitivity of components to RF excitation are of greatest value when the functioning level can be referred to the components' input terminals. Results indicating power delivered to a matched system of which the device is a component are of little value other than to give "ball park" information. Evaluations aimed at determining safe exposure levels may be founded on "power at the switch" tests, but for the purpose of the HERO test schedule we need to determine the best estimate of the power at the components' terminals. If we are to have a sound basis for comparing one item with another, we must be certain that we are not, by chance, comparing one matching system with another. Though we will be forced to continue testing on the basis of setting power input to a matched system, we must be certain to have at our disposal a technique for establishing the power which reaches the base of the plug.

More recently, in this program, two methods for measuring power delivered to the terminals of an arbitrary termination have come under close scrutiny. One is a dual-directional coupler technique and the other, a slotted line voltmeter technique. The voltage-impedance method is being examined on another program. It appears that these three will serve to facilitate power measurements from 1 Mc to 10 Gc.

Intensive study of the separate systems will reveal how well and over what range they may be expected to be effectively employed. It is within this framework that we have concentrated on examining the voltage min-max technique for measuring power during the present period. Progress on this is reported in the following sections.

1.1 Voltage Min-Max Measurements of Power to the Base of a MARK 1 MOD 0 Squib at 1 Gc

Power measurements at 3 Gc with a MARK 1 MOD 0 squib termination were described in our last report. Similar tests at 1 Gc, with the same termination, were made during the present period except that we did not make companion measurements with the differential power technique. Instead, we concentrated upon validating the voltage min-max method. Of all methods considered, this appeared to hold greatest promise for estimating the power entering the terminals of an EED load with the minimum amount of development.

The essential features of the system are shown in Figure 1-1. A Hewlett-Packard slotted line inserted between the adjustable line and the RF amount permits making the maximum and minimum voltage readings necessary to compute net power in the line. Fortunately, the slotted line had an insertion length very close to 60 centimeters, an even multiple of the wavelength corresponding to 1 Gc. This made it possible to match at the input of the system with the modified line stretcher which has an adjustment limited to only 3 centimeters. Other than the slotted line, this equipment is identical to that used to evaluate the

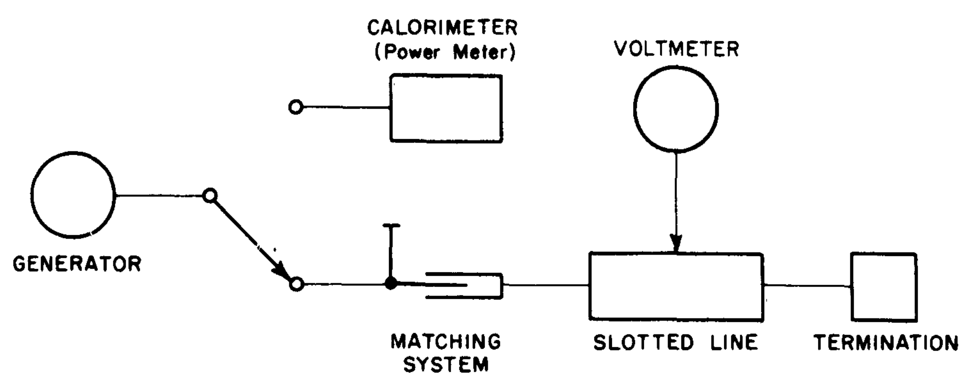


FIG. 1-1. SYSTEM FOR VOLTAGE MIN-MAX POWER MEASUREMENTS

MARK 1 MOD 0 Squib at 1 Gc on an earlier program. More will be said of this in Section 1.4.

All data from the test are given in Table 1-1. It will be noted that the net power and VSWR values shown in this table are shown as occurring at voltage minimum points only. We had originally thought that these values would be located midway between maximum and minimum points. However, further consideration of the initial data led to the conclusion that the effective locations of both the power and VWSR values determined by associating either adjacent voltage maximum with a voltage minimum were closer to the minimum point. A theoretical analysis in Section 2.1 of this report confirms this conclusion and predicts the exact location of the point of measurement. For VSWR's greater than 10 our conjecture is justified. Thus in Table 1-1, where a voltage maximum occurs on either side of a minimum, we have computed the net power and VSWR for each set and referred their average to the location of the voltage minimum. These data are plotted in Figure 1-2 showing the variation in measured net power as a function of distance from the load.

Three different values of system input power were used. We attempted to draw a straight line through the system input power and the measured net power points. We reasoned that extending this line to zero centimeters, the location of the load, would give an estimate of power at the base of the MARK 1 MOD 0 squib. When these base powers are compared with the power at the bridge wire, the loss in the base of the item is obtained. However, since these test data are not conclusive regarding actual value of power to the base no figures will be stated for probable base loss.

These results indicate, however, the expected trend, that successive power measurements along the length of the slotted line should give some indication of the loss introduced by the system components. Additional experiments to be discussed show this more clearly, and in addition point out probable causes for the uncertainty of these data.

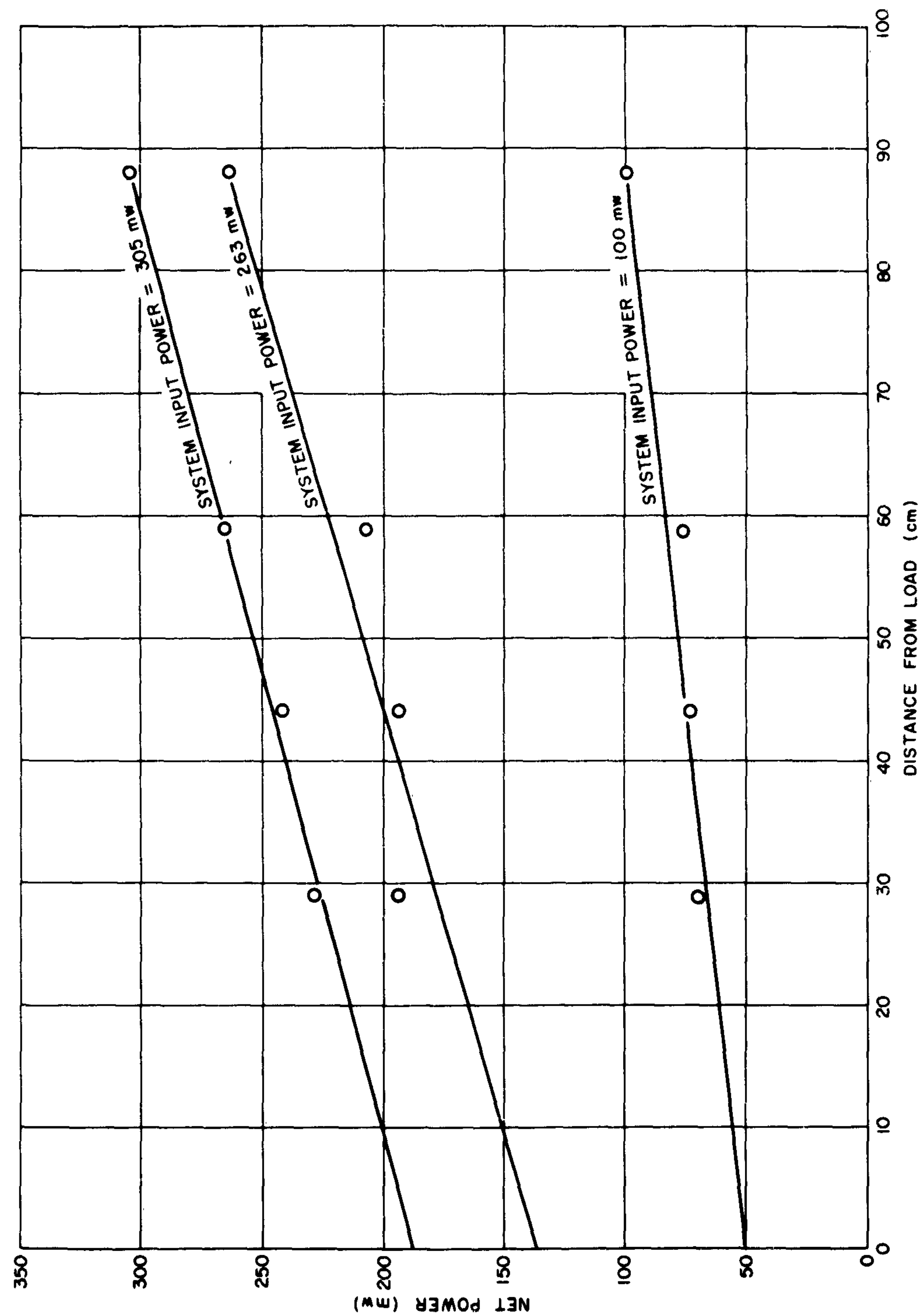


FIG 1-2. NET POWER VERSUS DISTANCE FROM LOAD
(Mark I Mod O Squib Termination)

Table 1-1

VOLTAGE MIN-MAX POWER MEASUREMENTS AT 1 Gc; MARK 1 MOD 0 TERMINATOR

System Input Power (watts)	Position of Probe in Slotted Line Centimeters from load	Probe Voltmeter Reading (volts)	Convert To True volts (volts)	Net Power $= \frac{V_{max} \times V_{min}}{50}$ (watts)	VSWR $= \frac{V_{max}}{V_{min}}$	Power At Bridge Wire-Indicated By Photo-Cell (watts)
0.305	29cm - min	0.004	0.478	0.2295	50.20	0.150 watts
	36cm - max	1.0	24.0			"
	44cm - min	0.0041	0.4825	0.2412	51.81	"
	51.5cm - max	1.05	26.0			"
	59cm - min	0.0046	0.512	0.2662	50.78	"
0.263	29cm - min	0.0038	0.465	0.1934	44.73	0.123 watts
	36cm - max	0.92	20.8			"
	44cm - min	0.0038	0.465	0.1934	44.73	"
	51.5cm - max	0.92	20.8			"
	59cm - min	0.0044	0.5	0.208	41.60	"
0.100	29cm - min	0.0014	0.28	0.070	44.65	-
	36cm - max	0.575	12.5			-
	44cm - min	0.00158	0.291	0.733	43.29	-
	51.5cm - max	0.585	12.7			-
	59cm - min	0.0016	0.30	0.0762	42.33	-

For example, in Table 1-1 it may be noted that voltages read at two minimum points separated by a half wave length are in some cases almost identical. This should not be, since the loss in this length of line would make some observable difference. The experience gained in subsequent tests suggests that faulty readings may be obtained at the minimum if the probe is not very carefully tuned. Tuning for both maximum and minimum should be done with care, though the greatest error will occur at the minimum points.

1.2 Slotted Line Probe Voltmeter Calibration

A calibration was necessary to obtain the data given above and that which will be discussed later. This was accomplished in the manner detailed in the previous report. In brief, the deflection of a voltmeter connected to the slotted line probe is noted for various power indications read on a calorimeter used as termination. Since the calorimeter's impedance matches the characteristic impedance of the line the voltage at any point on the line should be equal to $\sqrt{50P}$ where P is the calorimeter power indication in watts and 50 the system's resistive impedance in ohms, (reactance is zero for a flat system).

The range over which we must calibrate is dependent upon the anticipated SWR of the item to be evaluated. For the MARK 1 MOD 0 we had expected a VSWR on the order of 50, or an apparent power standing wave ratio of 2500. Our calibration therefore must span at least three decades of power, a greater span would give assurance that we had included the correct decades. It was on this latter score that we experienced difficulty, but engineering judgement led to a reasonable solution.

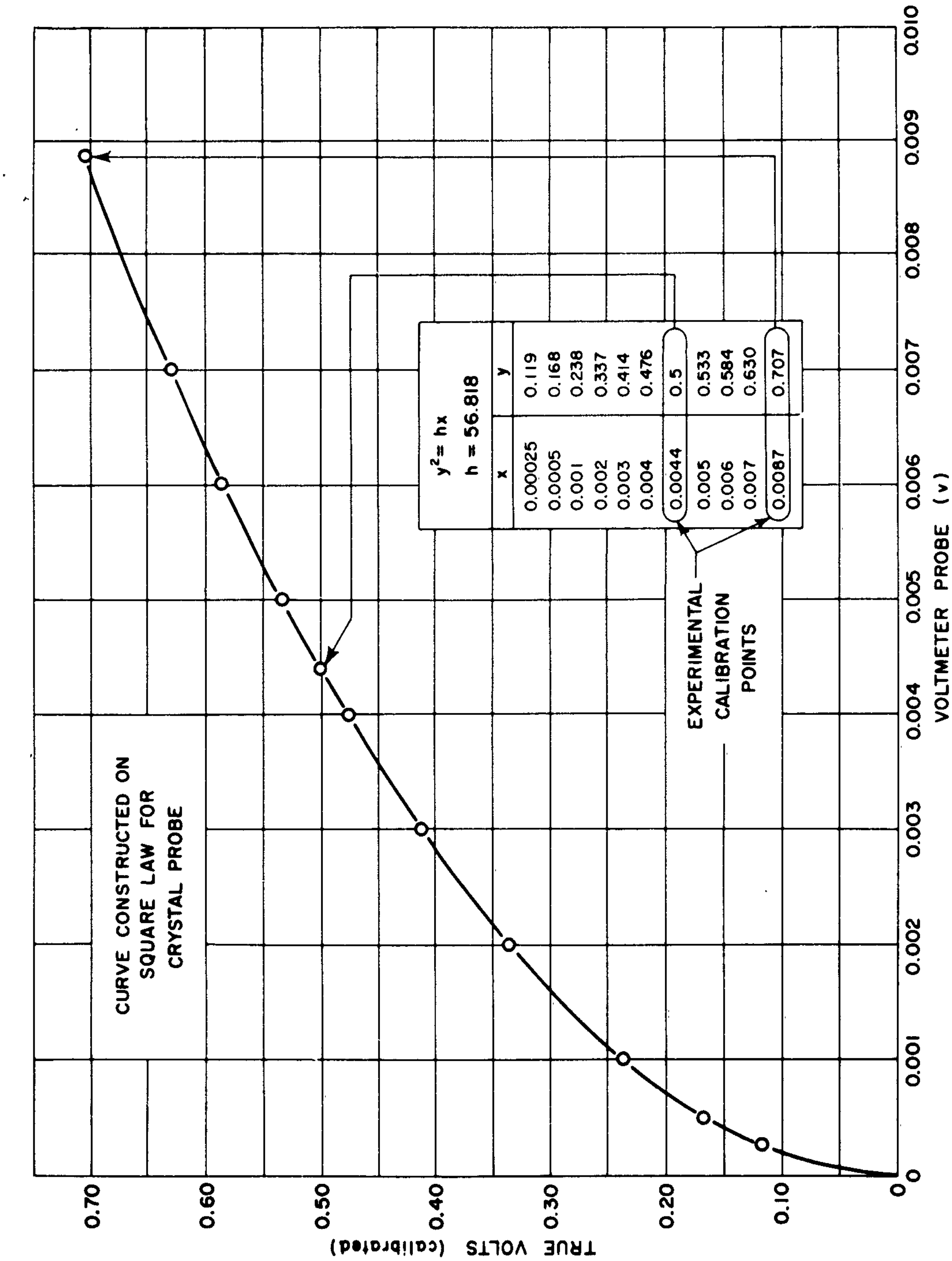
We began our calibration starting with a level of 10 watts and were successful in obtaining results down to 5 milliwatts. At this point, the calorimeter power meter was being operated in its most unstable range, and the uncertainty of readings made it impossible to

continue. Attempts to use the calibration curve made from these data showed that we needed more calibration points at the lower levels. Assuming that the probe voltmeter had a square law response at this level, we developed the synthetic calibration shown in Figure 1-3. The two low level calibration points and a point assumed at the origin were employed to fit the calculated curve to the experimental data. This curve is the calibration used for the data of Section 1.1; a similar synthetic calibration was made for the data to be discussed in the following section.

1.3 Voltage Min-Max Power Measurements with Disk Resistor Terminations at 1 Gc (Uniform Line)

The measurement system used to obtain the data given in Section 1.1 was assembled from components made of different types of transmission line and an assortment of connectors. For example, the Hewlett-Packard (HP) slotted line is basically a slab line, with short sections of coaxial line and type "N" connectors at the output and input ends. General Radio (GR) air line, used in other parts the circuit, is solid conductor coaxial transmission line. Each of these lines and associated connectors has a different loss factor. The slope of the power loss curve, therefore, can not be expected to be uniform throughout a system composed of different types of line. Verification of the technique is dependent upon our ability to uniquely define the system loss, a task made easier by limiting the variability in the system.

We decided to substitute a GR slotted line in place of the HP line and thus assemble a system using GR components only. Disk resistors with values of 0.5 and 1.0 ohm, mounted in a GR line system, were used for loads. Measurements made with this uniform system were expected to show a uniform loss. The loss is expressed at 1/2-wavelength points, since the loss is not uniformly distributed over the standing wave pattern. The conductor loss is greatest at current maximum points and least at current minimum. Shunt losses are similarly related to the voltage wave.



x	y
0.00025	0.119
0.0005	0.168
0.001	0.238
0.002	0.337
0.003	0.414
0.004	0.476
0.0044	0.5
0.005	0.533
0.006	0.584
0.007	0.630
0.0087	0.707

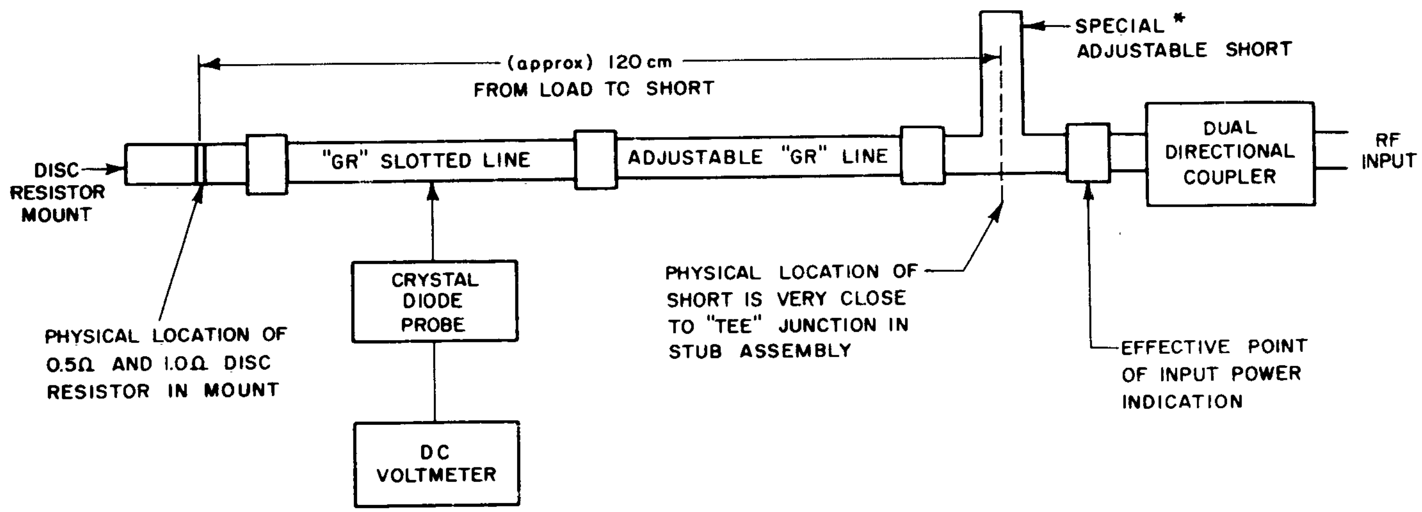
FIG. I-3. SYNTHETIC CALIBRATION OF PROBE AT 1Gc
(Hewlett-Packard Slotted Line)

The essential features of this measuring system are shown in Figure 1-4. The uniform line system begins at the physical location of the disk resistor termination and extends for approximately 120 centimeters to the location of the short circuit in the shorted stub. When the system is matched, the junction of the shorted stub and the adjustable line terminate the line in its characteristic impedance and the section of line from this point back to the generator is flat. However, relatively large standing waves exist on the 120 centimeters of line between the load and the shorted stub. Since the load is almost pure resistance the first voltage minimum occurs very near to the load. The first maximum will be displaced a quarter wavelength, or 7-1/2 cm. There were 6 maxima and 7 minima observed in the 120 centimeters of line with which we are concerned.

Referring again to Figure 1-4, measurements were first made with the slottled line located next to the load as shown. The positions of the slotted and adjustable line were then interchanged to obtain additional measurements at the other end of the system. The resulting data are given in Table 1-2. The net power and the VSWR at various distances from the load were calculated from these data and are included in the Table.

Figure 1-5 shows the change in net power as a function of the distance (in centimeters) from the load. The curves were expected to show a uniform slope over the entire 120 centimeter length. However, the curves indicate a discontinuity between data obtained for the two locations of the slotted line.

The variation of VSWR as a function of distance is given in Figure 1-6. A break is again seen in these curves. Indications are that a discontinuity, most probably due to a faulty section of adjustable air line, is accountable for this observation. Since all of the voltage tests were made on the slotted line itself and not in the



*NOTE

THIS SPECIAL ADJUSTABLE SHORT WAS MADE IN OUR LABORATORY SO THAT THE SHORT COULD BE LOCATED AT THE MINIMUM DISTANCE FROM THE JUNCTION OF THE STUB AND MAIN LINE. THIS DISTANCE LESS THAN A CENTIMETER FOR VERY LOW RESISTANCE LOADS.

FIG. I-4. UNIFORM LINE SET-UP FOR VOLTAGE MIN-MAX POWER MEASUREMENTS AT 1 Gc

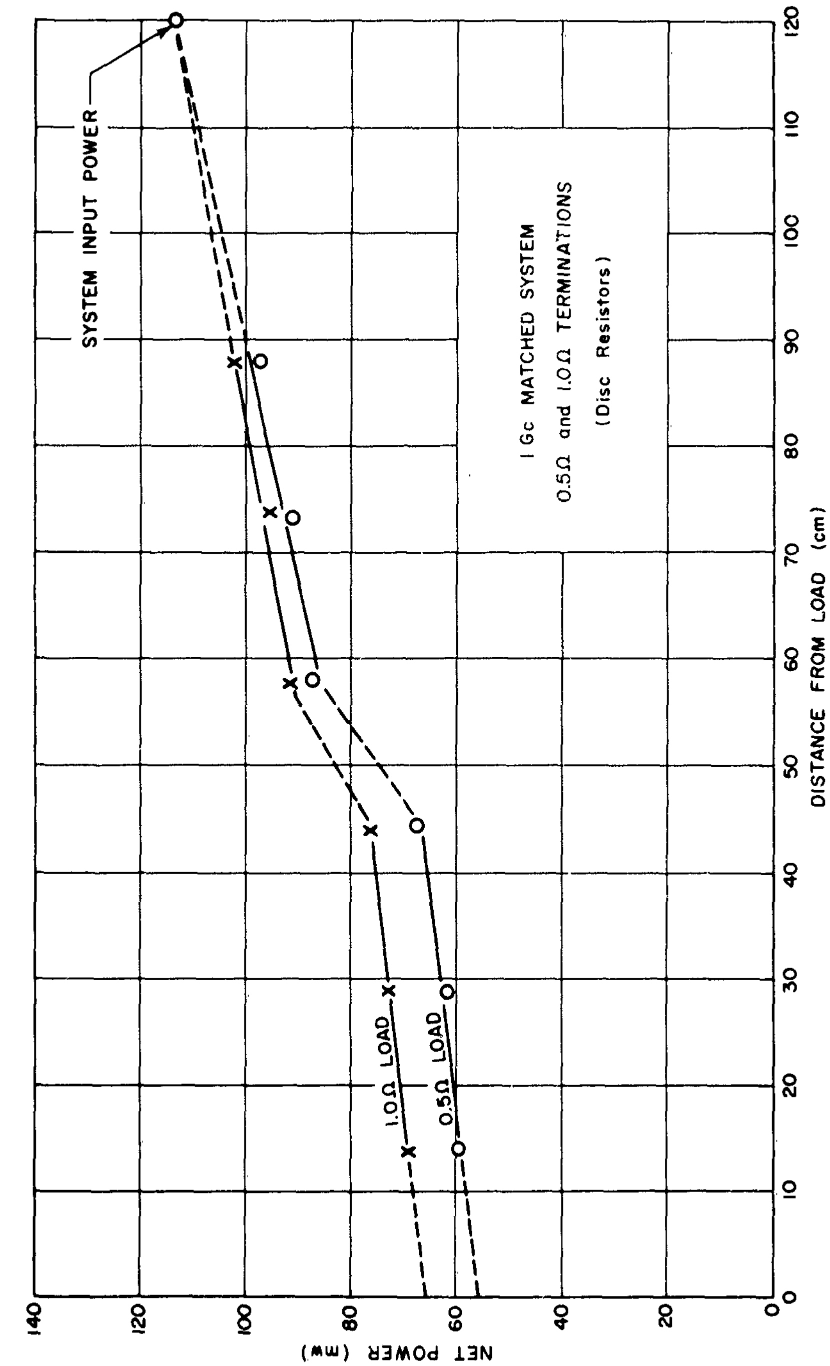


FIG. I-5. NET POWER VERSUS DISTANCE FROM LOAD

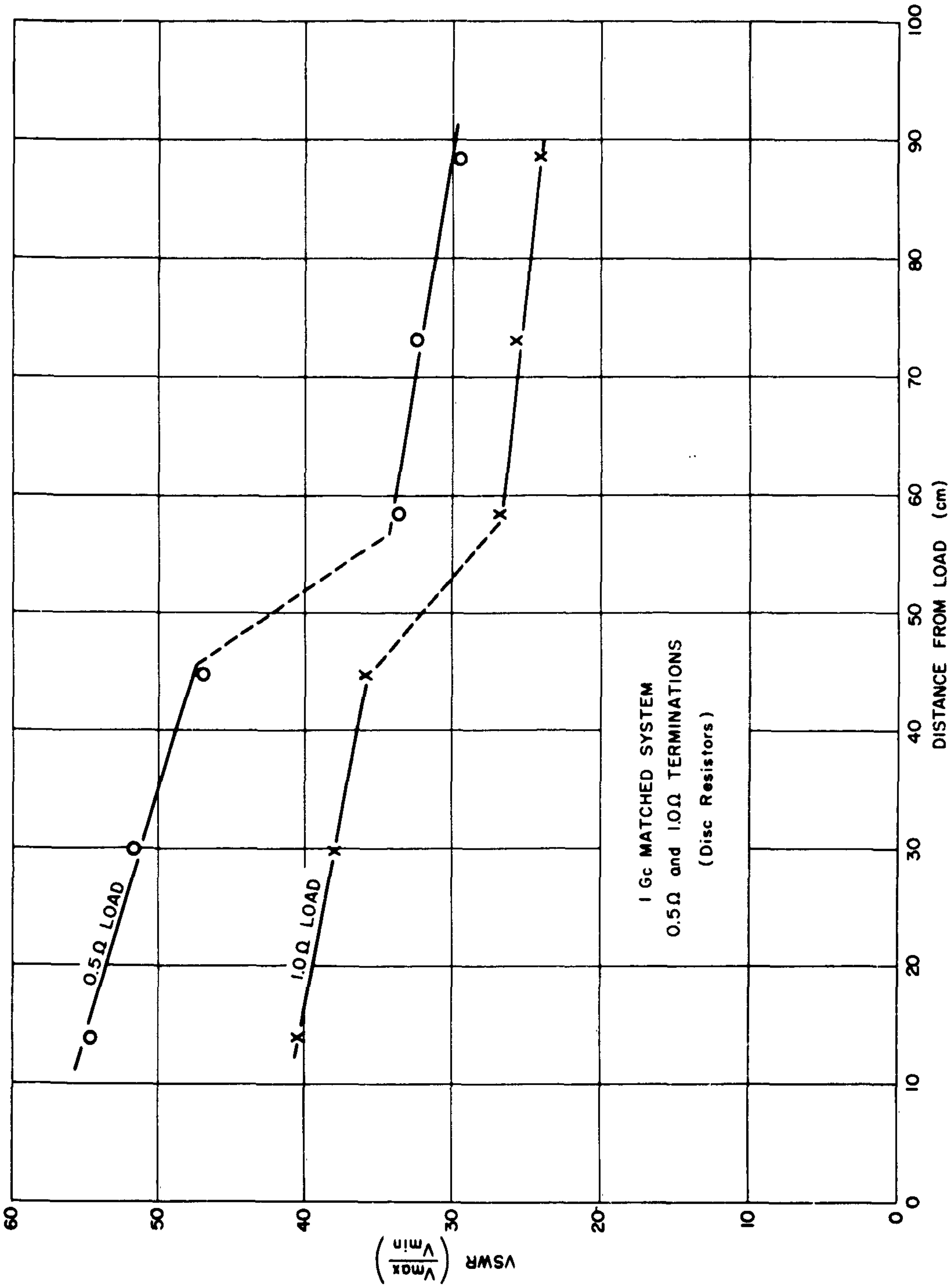


FIG. 1-6. VSWR VERSUS DISTANCE FROM LOAD

Table 1-2

MEASUREMENT OF POWER DELIVERED TO DISK RESISTOR TERMINATIONS

DC Resistance of load (ohms)	Distance From Load To Probe (Centimeter)	Probe Voltmeter Readings (volts)	True Volts (From Calibration) Curve	Net Power Toward Load $= \frac{V_{max} \times V_{min}}{50}$ (watts)	VSWR $= \frac{V_{max}}{V_{min}}$
0.5	14	0.00317	0.234	0.0599	54.70
0.5	21.5	1.025	12.8		
0.5	28.9	0.0035	0.245	0.06198	51.63
0.5	36.9	1.005	12.5		
0.5	44.9	0.00517	0.268	0.06775	47.16
0.5	51.8	1.0225	12.78		
0.5	59.	0.0054	0.307	0.07847	41.63
0.5	58.3	0.0072	0.360	0.08769	33.833
0.5	65.2	0.994	12.18		
0.5	23.3	0.00785	0.373	0.09071	32.60
0.5	80.7	0.990	12.14		
0.5	88.2	0.0089	0.394	0.09706	31.75
0.5	96.0	1.0007	12.5		
1.0	14	0.00493	0.292	0.06920	40.58
1.0	21.5	0.963	11.85		
1.0	28.9	0.0054	0.308	0.07223	38.06
1.0	36.9	0.950	11.6		
1.0	44.9	0.0060	0.325	0.07639	36.1
1.0	51.8	0.965	11.8		
1.0	59.0	0.0074	0.365	0.08687	32.6
1.0	58.3	0.0096	0.412	0.09146	26.9
1.0	65.2	0.920	11.1		
1.0	73.3	0.0101	0.425	0.09347	25.87
1.0	80.7	0.910	10.9		
1.0	88.2	0.0118	0.460	0.1026	24.24
1.0	96.0	0.940	11.4		

faulty section of line, the data obtained are still of value. However, the discontinuity has precluded a complete verification of the power measuring technique.

Results so far indicate that net power can probably be measured with a fair degree of accuracy by this means. Repeating these tests in a system which is not faulty should produce data to plot continuous curves. With a smooth curve of net power versus distance from the load we should be able to extrapolate to the location of the load, and thereby estimate the probable power level at the load.

If the smooth curve described above is obtained, validity of the technique will be clearly demonstrated. In addition, we are investigating the theoretical loss for the type of transmission line used. Preliminary checks have indicated a remarkable agreement between prediction and observation. The theoretical method is undergoing further study and results will be published when the tests have been repeated under more nearly ideal conditions.

1.4 Re-examination of System Calibration for 1 Gc Test of MARK 1 MOD 0 Squib

Precision Bruceton tests of the MARK 1 MOD 0 squib were made at a frequency of 1 Gc as part of an earlier program. Pertinent data is contained in report Q-B1805-5,* in which the conclusion is made that the base of the MARK 1 MOD 0 ignition element is essentially lossless at this frequency. Data collected on this current program, however, tends to indicate that there may be a reasonable loss in the base of this squib at 1 Gc. Accordingly we repeated the system efficiency measurement using the identical piping used in the earlier test. As stated before, the systems shown in Figure 1-1 and discussed in Section 1.1, with the exception of the slotted line, is the equipment reevaluated for system efficiency.

*Franklin Institute Report Q-B1805-5, Precision RF Sensitivity Studies (Evaluation of MARK 1 MOD 0 Squib and MARK 2 MOD 0 Ignition Element) by P.F. Mohrbach, R.F. Wood, and J.P. Warren, 1 Nov. 61 to 1 Feb. 62, Prepared for U.S. Naval Weapons Lab. under Contract No. N178-7830.

So as to determine the probable change in system efficiency due to the addition of extra piping, (the slotted line adds a full wavelength of pipe) an additional experiment was conducted. Results of this experiment indicate that the base of the MARK 1 MOD 0 Squib is probably not lossless.

In our earlier work, system efficiency was determined by comparing the RF and dc power required to produce a given bridge wire illumination. Bridge wire output was sensed by a photo-conductive detector mounted above the exposed bridge wire of an inert squib. A system efficiency of 51.4% was thus estabilised for the firing system used for the precision Bruceton firing tests of the MARK 1 MOD 0 squib at 1 Gc. The efficiency redetermined during this period confirmed the previously obtained results. The essential details of the system used are shown in Figure 1-7(a). The total length of the system, from the squib terminals to the position of the short in the shorted-stub, was approximately 30 centimeters, a full wavelength at 1 Gc.

It is well known that the addition of multiples of a half wavelength of transmission line, will not modify the rest of the system - if the line is lossless. If, however, the lines are not lossless, we would expect each added increment of line to add equal increments of loss to the system. We therefore reasoned that the addition of half-wavelength increments of line should require the system input power for a given output on the photo-detector to be increased by an amount proportional to the losses in the original 30 centimeters of line.

In the actual test, a 45-centimeter line was added; see Fig. 1-7(b). An 18% increase in system input power for the same detector output was required. This was much less than expected and suggests that some of the power previously attributed to system loss may in fact be lost in the base of the squib. The data from the test is given in Table 1-3.

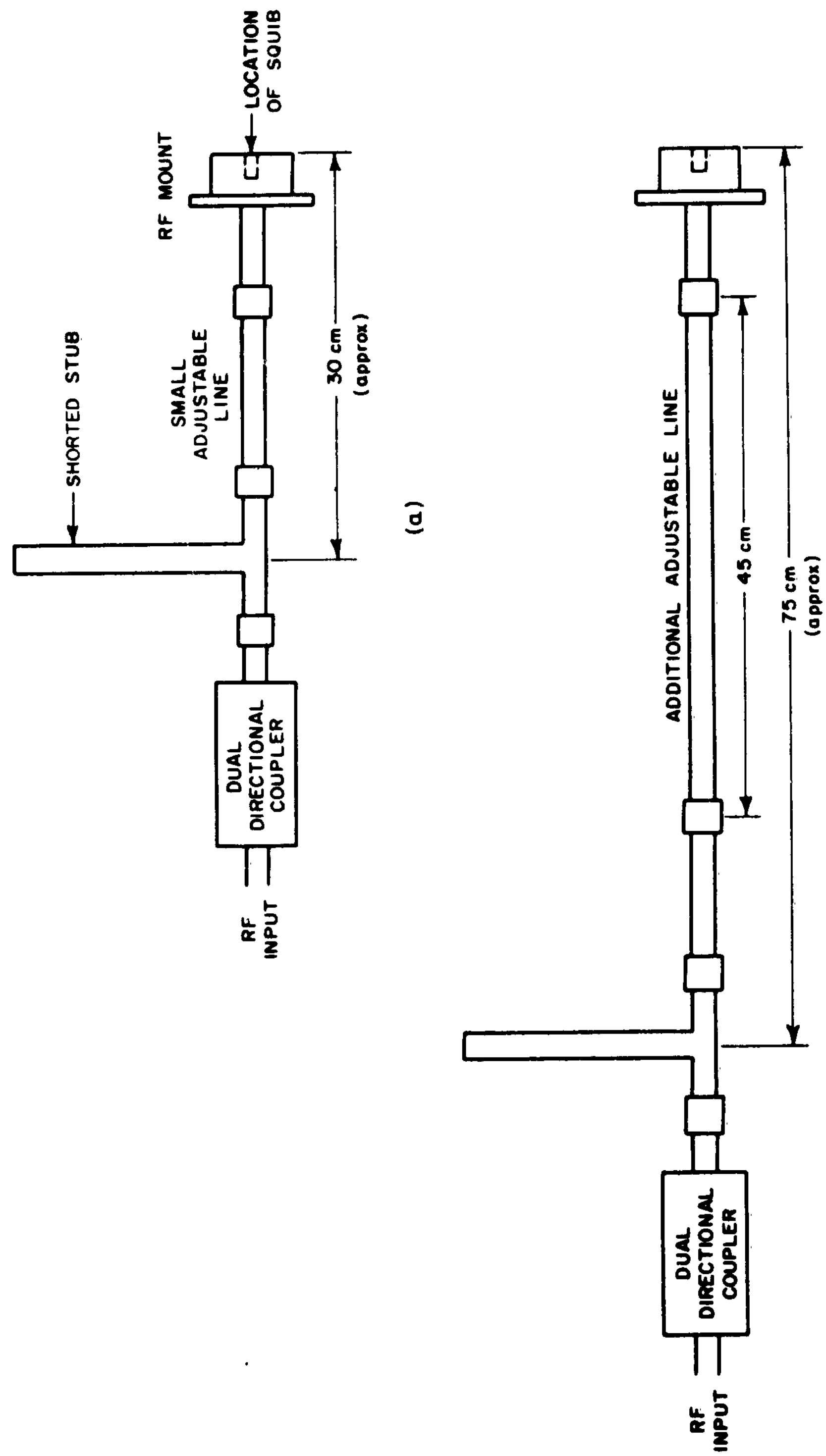


FIG.I-7. 1 Gc SYSTEM EFFICIENCY TEST SET-UP

Table 1-3

1 Gc SYSTEM EFFICIENCY DATA; MARK 1 MOD 0 SQUIB

System Input Power (watts)	Output of Clairex Cell (units)	DC Power Equivalent of Cell Output (watts)	Conditions of Test	System Efficiency Based on No Loss In Base %
0.275	25	0.150	Initial 30 cm System	54
0.220	6	0.113	Initial 30 cm System	51.3
0.325	25	0.150	Add 45 cm to Initial System	46.1
0.260	6	0.113	Add 45 cm to Initial System	43.5

Note

The average efficiency for an identical 30 cm system employed for an earlier test was 51.4% with a power input level of 0.220 watts.

The following argument for possible loss in the base is reasonable. Reference to Table 1-3 shows that with the initial 30-cm matching system, 0.275 watts of system input power was required for a detector output of 25 units. The system input power had to be raised to 0.325 watts, an increase of .050 watts, for the same detector output when the additional 45 cm of line was inserted. This would indicate that 0.0011 watt were lost per centimeter of pipe. According to this reasoning the original 30 cm matching system could be accountable for a loss of only .033 watts (.0011 x 30). With system input power equal to 0.275 watts, the difference, or 0.242 watts is thus the power at the

input or base of the squib. The photo-detector output indicates that approximately 0.150 watts reaches the bridge wire. The ratio $\frac{.150}{.242}$ therefore, corresponds to a power loss in the base of nearly 1.9 db.

A similar manipulation with the data obtained at the lower power level, corresponding to a detector output of 6 units (.113 watts dc), gives a loss factor of 0.00088 watts per centimeter of line. The loss in the original 30 cm of line follows as 0.0264 watts, and the power at the base of the squib becomes 0.1936 watts. The ratio of $\frac{.113}{.1936}$ corresponds to a base loss of roughly 2.2 db. The variation of 1.9 to 2.2 db for the two power levels is within expected experimental error.

While these results are certainly questionable, they nevertheless strongly suggests a reasonable value for the base loss. Results obtained with voltage-min-max power measurements also point to some base loss inherent in the MARK 1 MOD 0 squib. Additional work planned to verify the voltage probe - slotted line power measurement should lead also to the determinations of the magnitude of the loss.

2. THEORETICAL CONSIDERATIONS OF VOLTAGE STANDING WAVES ON LOW-LOSS TRANSMISSION LINE

In most RF systems, the assumption of a lossless line is valid in computation of performance. However, when one considers the characteristics of RF systems used in EED evaluation testing, "lossless line" analyses must be employed with extreme caution. For example, it may be argued that the VSWR on a transmission line is considered to be the same at any point along its length. Measurement of VSWR is, therefore, ordinarily made on a section of a supposedly lossless slotted transmission line, and many investigators would not question this measurement. The analyses of Section 2.2 will indicate to them some alarming considerations.

Power measurements by the voltage min-max method are of little value if we cannot specify the point to which the measurement applies. Consequently a theoretical analysis was conducted to determine this point

at which the computed power level is correct, in terms of the VSWR and the locations of the voltage maximum and minimum points. This analysis is given below. (In a lossless line, of course, the power level is the same throughout, and this question does not arise).

2.1 Effective Position of a Voltage Standing Wave Ratio Measurement on a Low-Loss Transmission Line

In a section of constant impedance transmission line that is terminated by an arbitrary impedance, both incident and reflected voltage waves and a resultant standing wave exist. The incident and reflected waves, V_i and V_r respectively, vary as functions of the distance along the line as given by

$$V_i = V_i\Big]_{x=0} e^{\alpha x} \tag{1}$$

$$V_r = V_r\Big]_{x=0} e^{-\alpha x} \tag{2}$$

where

V_i = RMS magnitude of the incident voltage wave (volt)
V_r = RMS magnitude of the reflected voltage wave (volt)
x = distance on the line (meter), positive toward the generator
α = attenuation constant for transmission line (neper/meter)

We shall define the voltage standing wave ratio S as

$$S = \frac{V_i + V_r}{V_i - V_r} \tag{3}$$

Since both voltage waves are functions of x, it follows that the voltage standing wave ratio must also be a function of x. Only for the case where the line has no losses ($\alpha = 0$) and the voltage waves have constant magnitudes does the voltage standing wave ratio become a constant. For this special case we can redefine S as

$$S = \frac{\left. V_{max} \right]_{x = q \pm \lambda/4}}{\left. V_{min} \right]_{x = q}} \tag{4}$$

where V_{max} is the in-phase addition and V_{min} is the out-of-phase addition (arithmetical difference) of the two waves. The position of V_{min} is designated by q. These voltage maxima and minima occur alternately along the transmission line at 1/4-wavelength intervals. With this formula S can be easily determined using a slotted line and voltage sensitive probe to measure V_{max} and adjacent V_{min}. Equation (4) is the one ordinarily used as the definition of the voltage standing wave ratio on a transmission line. It should be noted that it is valid only when the transmission line is lossless, and S is a constant value. Equation (3) represents the more general definition since it can be applied at any specified point on a lossy line.

Unfortunately the measurement of V_i and V_r at a specific position on a line is difficult to achieve, and it would be most convenient if we can adapt the slotted line technique, using Equation (4), to the lossy transmission line. This may be accomplished by equating a determination of S as given by Equation (4) to an equal standing wave ratio existing on the line as given by Equation (3). The derived expression should relate the actual position of S to either V_{max} or V_{min} in terms of the magnitude of S.

To make our work simpler we shall assume that the transmission line losses are very small, since this would be the case for a well designed slotted line. For $\alpha x \ll 1$.

$$e^{\pm \alpha x} = 1 \pm \alpha x \tag{5}$$

The higher order terms of this series expansion are omitted since they are negligible. Equations (1) and (2), which express the voltage waves as functions of x, can now be rewritten as

$$V_i = V_{i_o} + \alpha x \, V_{i_o} \tag{6}$$

$$V_r = V_{r_o} - \alpha x \, V_{r_o} \tag{7}$$

where V_{i_o} means $\left. V_i \right]_{x=0}$, and similarly for V_{r_o}.

Figure 2-1 represents the incident and reflected voltage waves on a low-loss transmission line. Since the waves are not of constant magnitude, the special-case formula, Equation (4), is no longer valid, and to find the value of S we must revert to the theoretical Equation (3). The difficulty then arises that we must make use of physically available measurements, of V_{max} and V_{min}, at different points; we must then reduce the readings to a true value of S at some point to be determined.

If we measure a V_{min} and an adjacent V_{max}, the ratio of V_{max}/V_{min} has the appearance of a standing wave ratio. Let us find the point where the true standing wave ratio has this value. For convenience, let us designate this point as the origin, $x = 0$. V_{min} is found at the point $x = q$, and V_{max} at $x = q \pm \lambda/4$, the signs depending on which side of V_{min} the reading of V_{max} was made.

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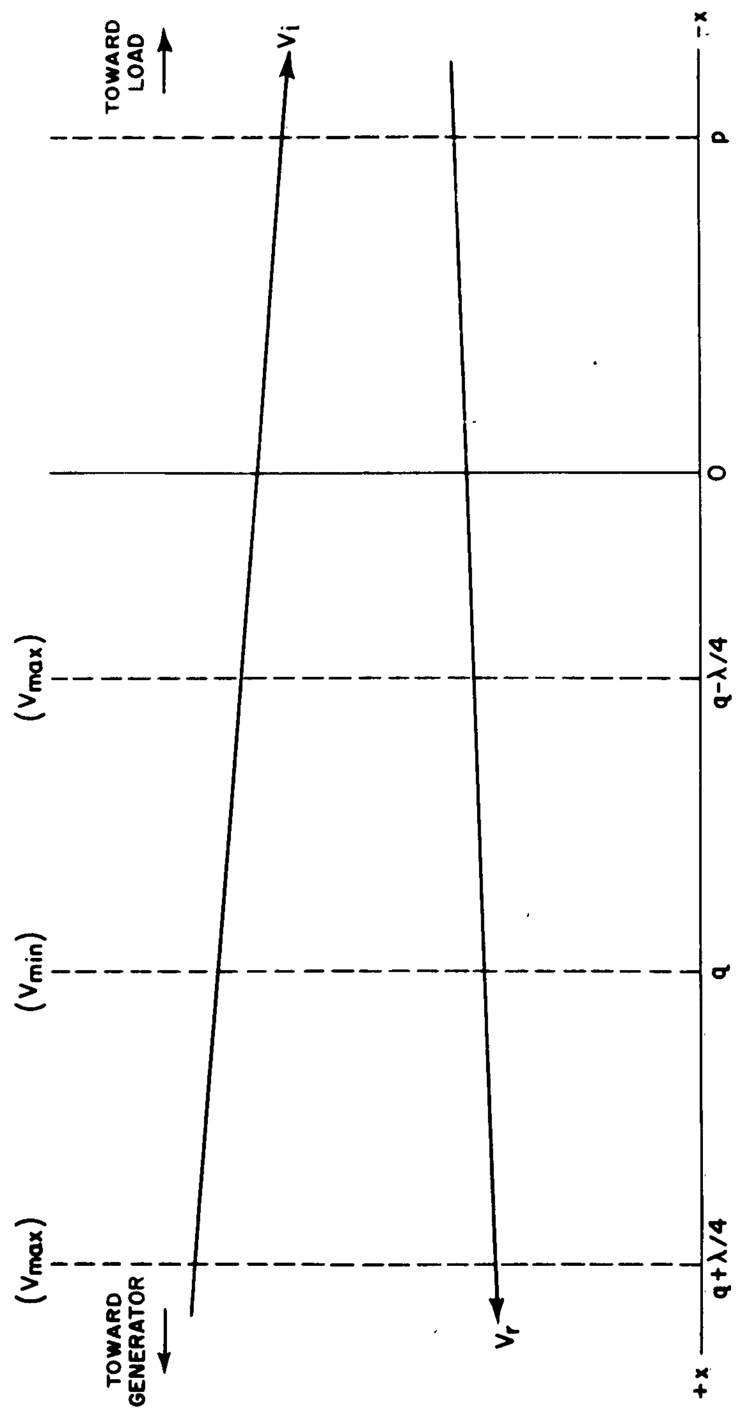

FIG. 2-1. GRAPHICAL REPRESENTATION OF VOLTAGE WAVES ON A LOW-LOSS TRANSMISSION LINE

Then

$$\frac{V_{max}\Big]_{x = q \pm \lambda/4}}{V_{min}\Big]_{x = q}} = S = \frac{V_{i_o} + V_{r_o}}{V_{i_o} - V_{r_o}} \tag{8-9}$$

If we can find the value of q, we then know the point at which the ratio of measurements is the true standing wave ratio.

The incident and reflected voltages at q and $q \pm \lambda/4$ are, from Equations (6) and (7),

$$V_i\Big]_{x = q} = V_{i_o} + \alpha q\, V_{i_o} \tag{10}$$

$$V_r\Big]_{x = q} = V_{r_o} - \alpha q\, V_{r_o} \tag{11}$$

$$V_i\Big]_{x = q \pm \lambda/4} = V_{i_o} + \alpha(q \pm \lambda/4)V_{i_o} \tag{12}$$

$$V_r\Big]_{x = q \pm \lambda/4} = V_{r_o} - \alpha(q \pm \lambda/4)V_{r_o} \tag{13}$$

Expanding Equation (8) into the form of Equation (3) and substituting Equations (10) through (13) we have

$$S = \frac{V_i\Big]_{x = q \pm \lambda/4} + V_r\Big]_{x = q \pm \lambda/4}}{V_i\Big]_{x = q} - V_r\Big]_{x = q}} \tag{14}$$

$$S = \frac{\left(V_{i_o} + \alpha(q \pm \lambda/4)V_{i_o}\right) + \left(V_{r_o} - \alpha(q \pm \lambda/4)V_{r_o}\right)}{\left(V_{i_o} + \alpha q\, V_{i_o}\right) - \left(V_{r_o} - \alpha q\, V_{r_o}\right)}$$

$$S = \frac{\left(V_{i_o} + V_{r_o}\right) + (q \pm \lambda/4)\left(V_{i_o} - V_{r_o}\right)}{\left(V_{i_o} - V_{r_o}\right) + \alpha q \left(V_{i_o} + V_{r_o}\right)}$$

Dividing through by $\left(V_{i_o} - V_{r_o}\right)$

$$S = \frac{\left(\dfrac{V_{i_o} + V_{r_o}}{V_{i_o} - V_{r_o}}\right) + \alpha(q \pm \lambda/4)}{1 + \alpha q\left(\dfrac{V_{i_o} + V_{r_o}}{V_{i_o} - V_{r_o}}\right)}$$

Substituting from Equation (8) and rearranging terms we finally get

$$q = \pm \frac{\lambda/4}{(S^2 - 1)} \tag{15}$$

This is an expression for the distance, in 1/4-wavelengths, from the position of S on the line to the position where V_{min} is measured. Several interesting facts can be obtained from this equation. The quantity $\lambda/4(S^2 - 1)$ while infinite when S equals unity rapidly approaches zero as S becomes greater than unity. See Figure 2-2. In fact when S becomes greater than 7, the effective position of S is just about equal to the position of V_{min}. For low values of S the following conclusions may be drawn. When V_{min} is closer to the load, Equation (15)

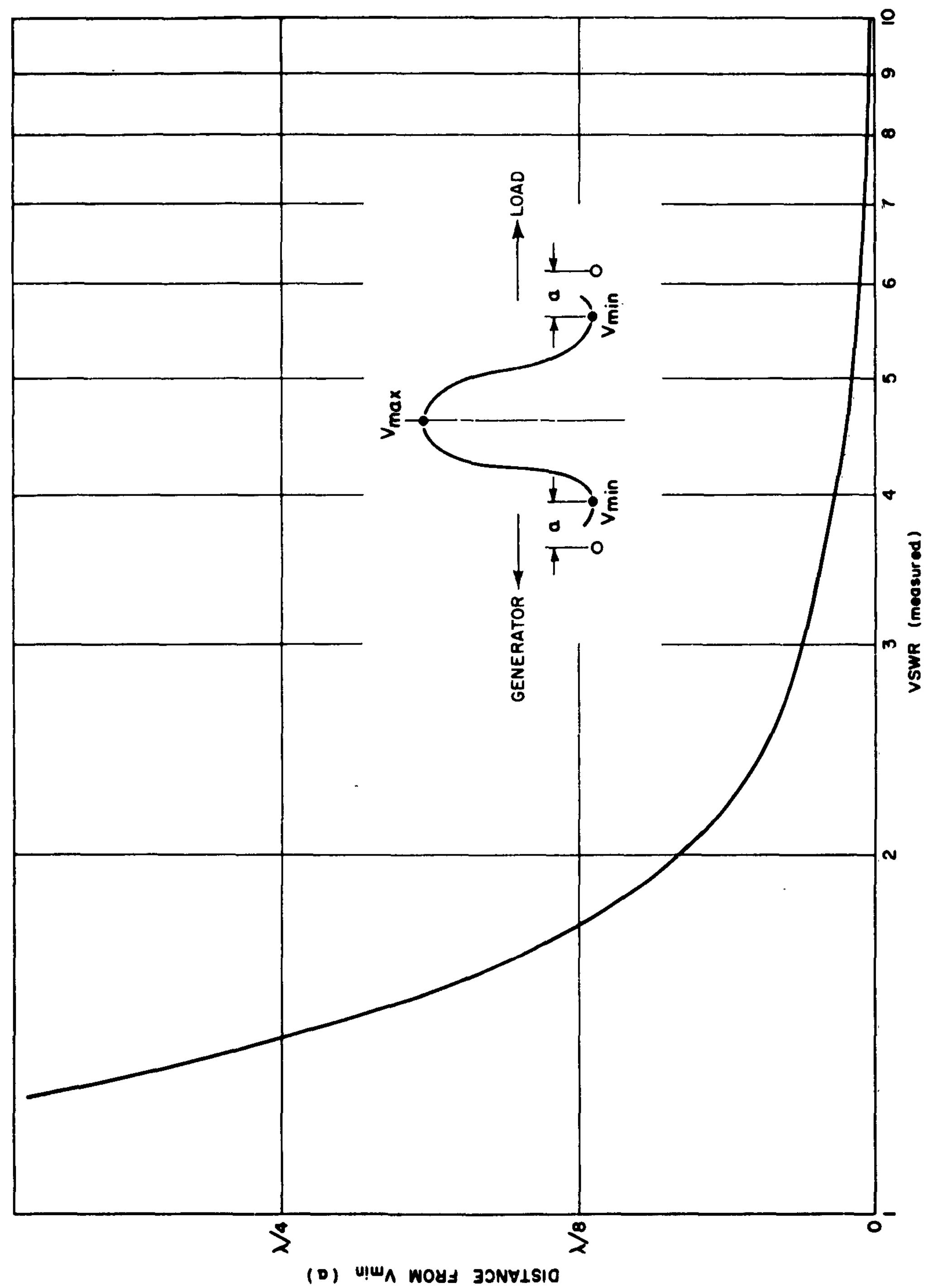

FIG 2-2. LOCATION OF MEASURED VSWR

is positive which means that the effective position of S is closer to the termination than either V_{max} or V_{min}. When V_{max} is closer to the load, Equation (15) is negative, which means that the effective position of S is further from the termination than either V_{max} or V_{min}. In either case the position of S does not fall between V_{max} and V_{min} as one might assume, but is always on the opposite side of V_{min} from whichever V_{max} is used, but only by a small distance. Finally, when S tends toward unity, q increases without limit. We cannot consider this situation, however, because our approximation that $\alpha x \ll 1$ is no longer valid.

We are now in a position to use slotted line techniques to determine the voltage standing wave ratio in a low-loss transmission line; using the approximation that for S greater than 7 we can assume the effective position of S on the line to be at the position where V_{min} is measured.

2.2 Change in Voltage Standing Wave Ratio Along Low-Loss Transmission Line

It is obvious, from what has gone before, that we cannot determine the standing wave ratio at the termination of a line solely from measurements made at that point; one, or probably both measurements must be made elsewhere. The attenuation of the line will cause the standing wave ratio computed from the measurements to be lower than the ratio at the termination.

We shall develop an expression which will give us the percent difference in these standing wave ratios as a function of both the total attenuation between the measurement position and some desired position (usually the termination), and also the magnitude of the standing wave ratio.

Figure 2-1 represents a section of low-loss transmission line terminated by an arbitrary load. We desire to compute S_p at any desired position x = p in terms of S determined for the point x = 0.

Incident and reflected voltages on the low-loss line were given in Equations (6) and (7); they are repeated here for reference.

$$V_i = V_{i_o} + \alpha x\, V_{i_o} \tag{6}$$

$$V_r = V_{r_o} - \alpha x\, V_{r_o} \tag{7}$$

where V_{i_o} and V_{r_o} are the voltages at the effective position x = 0, of the measured voltage standing wave ratio S. To find the standing wave ratio S_p at any other point x = p, we have

$$V_i\Big]_{x = p} = V_{i_o} + \alpha p\, V_{i_o} \tag{16}$$

$$V_r\Big]_{x = p} = V_{r_o} - \alpha p\, V_{r_o} \tag{17}$$

The standing wave ratio at x = 0 is $S = \dfrac{V_{i_o} + V_{r_o}}{V_{i_o} - V_{r_o}}$ (9)

At position p we can represent S_p by

$$S_p = \frac{\left(V_{i_o} + \alpha p\, V_{i_o}\right) + \left(V_{r_o} - \alpha p\, V_{r_o}\right)}{\left(V_{i_o} + \alpha p\, V_{i_o}\right) - \left(V_{r_o} - \alpha p\, V_{r_o}\right)} \tag{18}$$

Rearranging terms and combining with Equation (9)

$$S_p = \frac{S + \ p}{1 + \ pS} \tag{19}$$

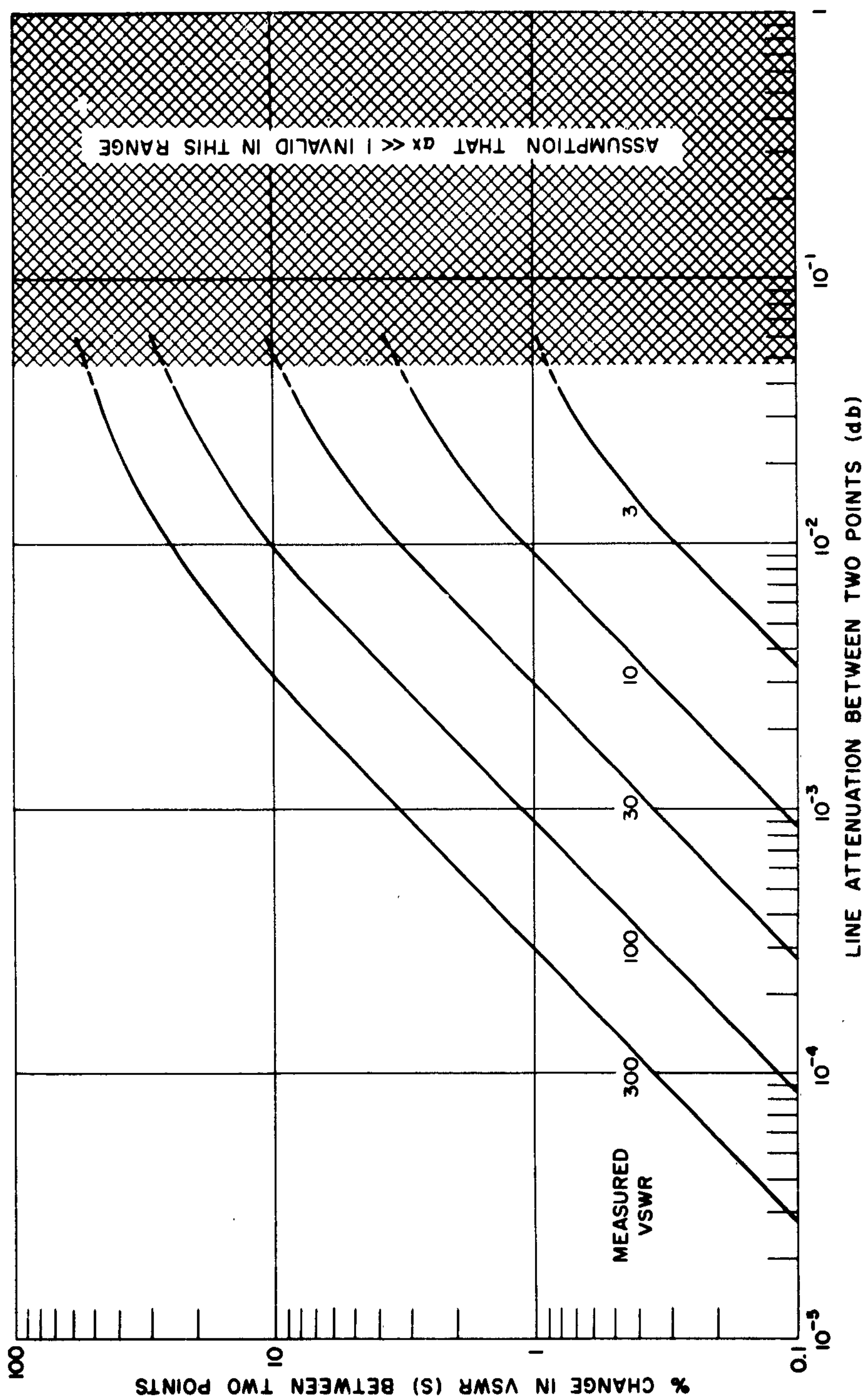

FIG. 2-3. MEASURED VSWR ERROR

Since line attenuation is usually presented in db/unit length, α can be converted by the expression

$$A = 8.68 \ \alpha (\text{db/unit length}) \tag{20}$$

Equations (19) and (20) produce the desired relationship. From these equations we have plotted a family of curves (Figure 2-3) representing different magnitudes of S as a function of the percent difference between S and S_p and as a function of the total attenuation between the two positions. This family of curves has several uses. First, it allows us to determine the correction factor needed when a standing wave ratio measurement is made at a remote distance from the desired position provided that the transmission line attenuation is known. It also has an additional use which may prove helpful to us. If two standing wave ratio measurements are made at different positions in a slotted line, the percent difference between the measurements can be calculated and the line attenuation between the effective positions of the measurements determined. This provides us with a novel way of measuring the attenuation on a low-loss transmission line using a slotted section.

Since the validity of equation (19) is based on the assumption that $\alpha x \ll 1$, then the total attenuation between points $x = 0$ and $x = p$ must be small.

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Paul F. Mohrbach
Group Leader

Norman P. Faunce
Project Leader

Approved by:

E.E. Hannum, Manager
Applied Physics Laboratory

Francis L. Jackson
Director of Laboratories

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